

Uttar Pradesh Rajarshi Tandon Open University

School of Science, Assignment Session 2021-22

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|------------------------|---------------------------------------|--------------------|
| Course Code: DCEMM-109 | Course Title: Abstract Algebra | Maximum Marks : 30 |
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(Section 'A')

(Long Answer Questions)

NOTE: Answer each question in 500 to 800 words. All carry equal marks.

Maximum Marks: 18

1. State and Prove fundamental theorem of group homomorphism.
2. Let N be a normal subgroups of a group G and H be a subgroup of G then show that:
(i) $H \cap N$ is normal subgroup of H (ii) HN is a subgroup of G (iii) N is normal subgroup of HN .
3. Prove that if G is abelian then $G/Z(G)$ is cyclic where $Z(G)$ is centre of G .

(Section – B)

(Short Answer Questions)

Maximum Marks: 12

Note: Answer each question in 200 to 300 Words. All carry equal marks.

4. Give all sub groups of $(\mathbb{Z}_{12}, +)$
5. Let $f: G_1 \rightarrow G_2$ be a group homomorphism then show that kernel f is a normal subgroup of G_1 .
6. Give an example non-cycle group whose all subgroups are cyclic.
7. Find all zero divisor elements of $\mathbb{Z}/20$.

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|------------------------|------------------------------------|--------------------|
| Course Code: DCEMM-110 | Course Title: Number Theory | Maximum Marks : 30 |
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(Section 'A')

(Long Answer Questions)

NOTE: Answer each question in 500 to 800 words. All carry equal marks.

Maximum Marks: 18

1. Find the remainders obtained on division of the following:
(a) 3^{50} by 101 (b) 159^{7654} by 23
2. Find the g.c.d. of 163 and 34 and express it in the form $163m + 34n$ in two ways.
3. Prove that (a) $18! + 1 \equiv 0 \pmod{437}$ (b) $28! + 233 \equiv 0 \pmod{899}$.

(Section – B)

(Short Answer Questions)

Maximum Marks: 12

Note : Answer each question in 200 to 300 Words. All carry equal marks.

4. Show that every square is congruent to 0 or 1 (mod 8).
5. Find the value of $\phi(m)$ if $m = 500$.
6. Find the following Legendre symbols: (a) $\left(\frac{19}{41}\right)$ (b) $\left(\frac{3}{7}\right)$ (c) $\left(\frac{5}{11}\right)$ (d) $\left(\frac{6}{11}\right)$
7. Find the value of Mobius function $\mu(n)$ for n
(a) 15 (b) 30 (c) 47 (d) 100

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| | | |
|------------------------|---------------------------------------|--------------------|
| Course Code: DCEMM-112 | Course Title: Advance Analysis | Maximum Marks : 30 |
|------------------------|---------------------------------------|--------------------|

(Section 'A')

(Long Answer Questions)

NOTE: Answer each question in 500 to 800 words. All carry equal marks.

Maximum Marks: 18

1. Every Cauchy sequence (S_n) of real Numbers converges.
2. Let (X_1, d_1) and (X_2, d_2) be two discrete metric spaces. Then verify that the product metric on $X_1 \times X_2$ is discrete.
3. Show that a Cauchy sequence is convergent \Leftrightarrow it has a convergent subsequence.
4. Let (X, d) be a metric space and $A \subseteq X$. Show that $\bar{A} = \{x \in X : d(x, A) = 0\}$.

(Section – B)

(Short Answer Questions)

Maximum Marks: 12

Note : Answer each question in 200 to 300 Words. All carry equal marks.

5. Define Complete Metric Space. Given an example of a metric space which is not Complete.
6. Any compact metric space is totally bounded.
7. Statement and Prove Mean value theorem.

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| | | |
|------------------------|---|--------------------|
| Course Code: DCEMM-113 | Course Title: Function of Complex Variable | Maximum Marks : 30 |
|------------------------|---|--------------------|

(Section 'A')

(Long Answer Questions)

NOTE: Answer each question in 500 to 800 words. All carry equal marks.

Maximum Marks: 18

1. If $u = \frac{1}{2} \log(x^2 + y^2)$, find v such that $f(z) = u + iv$ is analytic. Determine $f(z)$ in terms of z .

2. Find the radius of convergence R of the following power series:

(i) $\sum_{n=0}^{\infty} z^n$ (ii) $\sum_{n=1}^{\infty} \frac{z^n}{n}$ (iii) $\sum_{n=1}^{\infty} \frac{z^n}{n^2}$

3. Using Cauchy integral formula, calculate the following integrals.

$\int_C \frac{\cos(\pi z)}{z(z^2+1)} dz$, where C is the circle $|z| = 2$

4. Evaluate $\int_0^{3+i} z^2 dz$ along the line joining the points $(0, 0)$ and $(3, 1)$.

(Section – B)

(Short Answer Questions)

Maximum Marks: 12

Note : Answer each question in 200 to 300 Words. All carry equal marks.

5. Evaluate $\int_C \frac{dz}{z-2}$ for $n = 2, 3, 4 \dots$ where $z = a$ is a point inside the simple closed curve c .

6. Find Taylor Series of $f(z) = \frac{1}{z}$ about $z = -1, z = 1$ and $z = 2$. Determine the circle of convergence in each case.
7. For the conformal transformation $w = z^2$. Show that the circle $|z - 1| = 1$ transforms into the cardioid $R = 2(1 + \cos\theta)$ where $w = Re^{i\theta}$ in the w -plane.

Uttar Pradesh Rajarshi Tandon Open University

School of Science, Assignment Session 2021-22

Course Code: SBSMM-03

Course Title: **Elementary Analysis**

Maximum Marks : 30

(Section 'A')

(Long Answer Questions)

NOTE: Answer each question in 500 to 800 words. All carry equal marks.

Maximum Marks: 18

1. Write truth tables for the sentence $P \Rightarrow P$ and

$P \Rightarrow \neg P$. Is the first sentence a tautology.

2. The diagonal or the equality relation & *in a set S is an equivalence*

relation in S. For it $x, y \in S$ the $x \sim y$ iff $x = y$.

3. Let X be a set. Consider the relation R in $(\mathcal{P}(X))$, given by : for A, B

$A \sim B$ if $A \subseteq B$.

4. Let $f: X \rightarrow Y$ be a map and let A and B subsets of X , then $A \subseteq B \Rightarrow f(A)$

$\subseteq f(B)$

(Section – B)

(Short Answer Questions)

Maximum Marks: 12

Note : Answer each question in 200 to 300 Words. All carry equal marks.

5. Let $X = \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$, $Y = [-1, 1]$

Let $f: X \rightarrow Y$ given by $f(x) = \sin x$, $x \in X$.

6. Evaluate $\iint xy \, dx \, dy$ over the region in the positive quadrant for which $x + y \leq 1$.

7. Find the volume inside the paraboloid $x^2 + 4z^2 + 8y = 16$ and on the positive side of xz -plane.

