

Uttar Pradesh Rajarshi Tandon Open University

School of Science, Assignment Session 2021-22

Course Code: UGMM -101	Course Title: Differential Calculus	Maximum Marks : 30
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(Section 'A')

(Long Answer Questions)

NOTE: Answer each question in 500 to 800 words. All carry equal marks.

Maximum Marks: 18

1. Let f be defined on \mathbb{R} such that $f(x) = 0$ and $f(x) = \frac{e^{1/x}}{1+e^{1/x}}$ when $x \neq 0$
Does *limt exit* when $x \rightarrow 0$
2. Let f be defined on \mathbb{R} such that $f(x) = 5x - 4$ when $0 \leq x \leq 1$
 $f(x) = 4x^2 - 3$ when $1 \leq x \leq 2$
 $f(x) = 5x + 4$ when $x > 2$
is f continuous at $x = 1$ and $x = 2$?
3. Show that if facion is differentiable at given point then it is continuous at that point. is the converse true ? Support your answer.

(Section – B)

(Short Answer Questions)

Maximum Marks: 12

Note : Answer each question in 200 to 300 Words. All carry equal marks.

4. Let R be a relation defined in the set of natural numbers \mathbb{N} such that
 $R = \{(x, y): 3x + y = 15\}$ find the domain and range of R .
5. Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be a map defined by $f(x) = x^2$ and
let $A = \{x \in \mathbb{R} : 1 \leq x \leq 2\}$ find $f(A)$
6. If $fx = 2x - 1$ and $g(x) = x + 4$ then find $(f \cdot g)(x)$.
7. Consider a map $f: \mathbb{R} \rightarrow \mathbb{R}$ such that $f(x) = 4x^2 - 3$ is f injective.

Uttar Pradesh Rajarshi Tandon Open University

School of Science, Assignment Session 2021-22

Course Code: UGMM-102	Course Title: Analytical Geometry	Maximum Marks : 30
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(Section 'A')

(Long Answer Questions)

NOTE: Answer each question in 500 to 800 words. All carry equal marks.

Maximum Marks: 18

1. Find the point of intersection of the line $\frac{x+1}{1} = \frac{y+3}{3} = \frac{z-2}{2}$ with the plain $3x + 4y + z = 10$
2. Find the equation of the sphere for which the circle $x^2 + y^2 + z^2 + 7y - 2z + 2 = 0$, $2x + 3y + 4z = 8$ is a great circle.
3. Find the equation of the tenant plains of the sphere $x^2 + y^2 + z^2 - 2x + 4y - 6z + 30 = 0$ which are parallel $2x - y + z = 0$

(Section – B)

(Short Answer Questions)

Maximum Marks: 12

Note : Answer each question in 200 to 300 Words. All carry equal marks.

4. *If the equation $x^2 - y^2 - 2x + 2y + \lambda = 0$ represent a degenerate conic then find the value of λ*
5. Find the angle between the pair of straight lines $x^2 + 4y^2 - 7xy = 0$
6. Find the perpendicular distance from the origin to the plain $x + 2y + z = 3$ also find the direction cosines of the normal to the plain.
7. Find the angle between the planes $2x - y + z = 5$ and $x + 3y + 2z = 7$

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Course Code: UGMM-103	Course Title: Integral Calculus	Maximum Marks : 30
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(Section 'A')

(Long Answer Questions)

NOTE: Answer each question in 500 to 800 words. All carry equal marks.

Maximum Marks: 18

1. Show that $xy = 1$ and $x^2 + y^2 = 2$ touch each other at two points.
2. Under what condition the curves $a_1x^2 + b_1y^2 = 1$ and $a_2x^2 + b_2y^2 = 1$ cut orthogonally
3. Find the angle of the intersection of the curves $y^2 = x$ and $x^2 + y^2 = 4$

(Section – B)

(Short Answer Questions)

Maximum Marks: 12

Note : Answer each question in 200 to 300 Words. All carry equal marks.

4. Show that $\int_0^{\pi/2} (\sin^2 x) \cos x \, dx = \frac{1}{3}$
5. Integrate $e^{ten x} \cdot \sec^2 x$ w.r.t. x
6. Evaluate $\int_0^{\pi/4} (ten^5 x) dx$
7. Integrate $\frac{\sqrt{x}}{1+x^{1/4}}$ w.r.t. x

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School of Science, Assignment Session 2021-22

Course Code: UGMM-104	Course Title: Differential Equation	Maximum Marks : 30
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(Section 'A')

(Long Answer Questions)

NOTE: Answer each question in 500 to 800 words. All carry equal marks.

Maximum Marks: 18

1. Solve that differential equation

$$(1 + 4xy + 2y^2)dx + (1 + 4xy + 2x^2) dy = 0$$

2. Solve $x^2 + p^2x = yp$
3. Find the orthogonal trajectories of the cardioid $r = a(1 - \cos \theta)$, a being the parameter.

(Section – B)

(Short Answer Questions)

Maximum Marks: 12

Note : Answer each question in 200 to 300 Words. All carry equal marks.

4. Solve $\frac{dy}{dx} = e^{x-y} + x^2e^{-y}$

5. Solve $x.Dy + y = xy^3$

6. Solve $y = cx + a/c$

7. Is the following equation exact $(1 + e^{x/y})dx + e^{x/y}(1 - x/y)dy = 0$

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School of Science, Assignment Session 2021-22

Course Code: UGMM-105	Course Title: Mechanics-I (Statics and Dynamics)	Maximum Marks : 30
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(Section 'A')

(Long Answer Questions)

NOTE: Answer each question in 500 to 800 words. All carry equal marks.

Maximum Marks: 18

1. If T be the tension at any point P of a common catenary and T_0 be the tension at the lowest point A then prove that $T^2 - T_0^2 = W^2$ when W in the weight of the arc AP of the catenary.
2. Five weightless rods of equal length are joined together so as to form a rhombus ABCD with one diagonal BD. A weight W be attached to C and the system be suspended from A then show that there is a thrust in BD equal $w/\sqrt{3}$.
3. The velocities of a particle along and perpendicular to the radius vector from a fixed point are $\propto r$ & $\mu\theta$. Find the path of the particle.

(Section – B)

(Short Answer Questions)

Maximum Marks: 12

Note : Answer each question in 200 to 300 Words. All carry equal marks.

4. A particle is allowed to move from the top of a cycloid whose vertex is upward and plane vertical with negligible velocity. Find the point where the particle leaves the cycloid.
5. A body consisting of a cone and a hemisphere on the same base rests on a rough horizontal table the hemisphere being in contact with the table of the height of the cone is $\sqrt{3}$ times the radius of the hemisphere. Find whether the equilibrium will be stable or unstable.
6. A particle moves with a central acceleration which varies inversely as the cube of the distance if it is projected from an apse at a distance a from the origin with velocity which is $\sqrt{2}$ time of the velocity for a circle of radius a then show that its path is $r \cos \frac{\theta}{\sqrt{2}} = a$.
7. A particle whose mass is m is acted upon by a force $m\mu \left(x + \frac{a^4}{x^3}\right)$ towards the origin if it starts from rest at a distance a then show that it will arrive at the origin in time $\frac{\pi}{4\sqrt{\mu}}$

Uttar Pradesh Rajarshi Tandon Open University

School of Science, Assignment Session 2021-22

Course Code: UGMM-106	Course Title: Mechanics-II (Dynamics and Hydrodynamics)	Maximum Marks : 30
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(Section 'A')

(Long Answer Questions)

NOTE: Answer each question in 500 to 800 words. All carry equal marks.

Maximum Marks: 18

1. Find the moment of inertia of a rod of length $2a$ & mass M about a line through its centre perpendicular to *its* length.
2. Find the moment of inertia of a circular disc of radius ' a ' about *its* diameter.
3. At the vertex c of a triangle ABC which is a right angle at c show that the principal axis in the plane are inclined to the sides at an angle $\frac{1}{2} \tan^{-1} \frac{ab}{a^2-b^2}$.

(Section – B)

(Short Answer Questions)

Maximum Marks: 12

Note : Answer each question in 200 to 300 Words. All carry equal marks.

4. One end of a light string is fixed to a point of the rim of a uniform circular disc of radius ' a ' & mass ' m ' and the string is wound several times round the rim. The free end is attached to a fixed point and the disc is held so that the part of the string not in contact with the vertical of the disc be let go find the acceleration & tension of the string.
5. Find the moment of inertia of a right circular cylinder about a straight line through its centre of gravity perpendicular to its axis.
6. A straight uniform rod can turn freely about one end O , hangs from O vertically. Find the least angular velocity with which it must begin to move so that it may perform complete revolution in a vertical plane.
7. Show that the moment of inertia of the area bounded by $r^2 = a^2 \cos 2\theta$ about its axis is

$$\frac{Ma^2}{16}(\pi - 8/3)$$

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School of Science, Assignment Session 2021-22

Course Code: UGMM-107	Course Title: Linear Algebra	Maximum Marks : 30
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(Section 'A')

(Long Answer Questions)

NOTE: Answer each question in 500 to 800 words. All carry equal marks.

Maximum Marks: 18

1. Find all eigen values and eigen vectors of a linear transformation
 $T : \mathbb{R}^3 \rightarrow \mathbb{R}^3$, defined as $T(x, y, z) = (2x + y, y - z, 2y + 4z)$. Is T diagonalizable
2. If w_1 and w_2 are any two finite subspaces of a vector space V then show that
$$\dim(w_1 + w_2) = \dim w_1 + \dim w_2 - \dim(w_1 \cap w_2)$$
3. Find the eigen Values and eigen vectors of the matrix $A = \begin{pmatrix} 1 & 1 & 3 \\ 3 & 2 & 4 \\ 3 & 4 & 5 \end{pmatrix}$

(Section – B)

(Short Answer Questions)

Maximum Marks: 12

Note : Answer each question in 200 to 300 Words. All carry equal marks.

4. Let V be a vector space over a field F such that it has no proper subspace. Then show that either
 $V = \{0\}$ or $\dim V = 1$.
5. Which of the following is a linear transformation where $T : \mathbb{R}^2 \rightarrow \mathbb{R}^2$
(a) $T(x_1, x_2) = (1 + x_1, x_2)$
(b) $T(x_1, x_2) = (x_2, x_1)$
6. A function f is defined on \mathbb{R}^2 as follows:
 $f(x, y) = (x_1 - y_1)^2 + x_1 y_2$, where $x = (x_1 - x_2)$ and $y = (y_1, y_2)$
Is f a bilinear forms? Verify.

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School of Science, Assignment Session 2021-22

Course Code: UGMM-108	Course Title: Calculus of function of several variable and Vector Calculus	Maximum Marks : 30
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(Section 'A')

(Long Answer Questions)

NOTE: Answer each question in 500 to 800 words. All carry equal marks.

Maximum Marks: 18

1. at $u = e^{xyz}$ then show that $\frac{\partial^3 u}{\partial x \partial y \partial z} = (1 + 3xyz + x^2 y^2 z^2)$ is it also equal to $\frac{\partial^3 u}{\partial y \partial z \partial x}$?
2. Show that $\frac{\partial(u,v)}{\partial(x,y)} \times \frac{\partial(x,y)}{\partial(u,v)} = 1$
3. A particle moves so that its position vector is given by $\vec{r} = \hat{i} \cos wt + \hat{j} \sin wt$ Show that the velocity \vec{v} is perpendicular \vec{r} and $\vec{r} \times \vec{v}$ is constant vector.

(Section – B)

(Short Answer Questions)

Maximum Marks: 12

Note : Answer each question in 200 to 300 Words. All carry equal marks.

4. Find the deviational derivative of $f(x) = xy^2 + yz^3$ at the point $(1, -1, 1)$ along the vector $\hat{i} + 2\hat{j} + 2\hat{k}$
 5. at $u = \tan^{-1}\left(\frac{x^3+y^3}{x-y}\right)$ then show that $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \sin 2u$
 6. Determine the point where the function $x^4 + y^4 - 2x^2 + 4xy - 2y^2$ has a maximum or minimum.
 7. Find curl (curl \vec{F}) at the point $(0, 1, 2)$ where $\vec{F} = (x^2 y)\hat{i} + (xyz)\hat{j} + (z^2 y)\hat{k}$
- Or
- Evaluate $\int \vec{F} \cdot d\vec{r}$ where $\vec{F} = (3x^2)\hat{i} + (2xz - y)\hat{j} + z\hat{k}$ along the straight line joining $(0, 0, 0)$ & $(2, 1, 3)$